

- DISPERSION

Using antennas, radio stations transmit radio waves. The waves they transmit travel at the speed of light ("c") and have "frequencies". The frequency indicates the number of times per second the wave vibrates. By law, each radio station is assigned a specific frequency. A light source also transmits waves called light rays. Unlike a radio station which transmits only a single frequency, however, a light source transmits a range of frequencies. If we extend the tuning band of a radio to include extremely high frequencies, we eventually arrive at the "light band". In this band, each different frequency represents a different color. Indeed, just as each radio station is identified by its radio frequency, each color is identified by its frequency. Red light, for example, has a frequency $4.6 \times 10^{14} \mathrm{~Hz}$, blue light has a frequency of $6.5 \times 10^{14} \mathrm{~Hz}$, etc. In effect, our eyes act as a radio receiver capable of receiving (seeing) all the light frequencies (colors).

Consider a beam of white light (a mixture of all the colors) incident on a prism. Each color refracts at a slightly greater angle upon entering the prism. When the colors emerge, again each refracts at a slightly greater angle. The net effect is that white light is separated into its constituent colors. This process of the colors spreading out is known as dispersion. The diagram below illustrates this effect. The band of colors (identical to the colors of a rainbow) is known as a spectrum.


The rainbow is produced by the combined effects of refraction, dispersion and total internal refraction by the droplets of water in the air. Under the right conditions, two rainbows are visible, an inner and an outer bow. The inner bow is much brighter than the outer bow. Moreover, the inner bow is red on the outside and violet on the inside while in the outer bow the colors are reversed.

## - THE CRITICAL ANGLE

The critical angle of an optical (transparent) medium is the limiting angle of incidence that results in an angle of refraction equal to $90^{\circ}$.
${ }^{4}$ Note: Given the index of refraction of an optical medium, we can use Snell's Law to calculate the critical angle. In using the formula, the angle of refraction is set to $90^{\circ}\left(r^{\circ}=90^{\circ}\right)$.

## HISTORICAL NOTE

Rene Descartes was the first to publish the law of refraction in 1637. The reason the law is called Snell's Law, however, is because of jealousy on the part of the English and the Dutch who refused to credit the discovery to someone from France.

## - TOTAL INTERNAL REFLECTION

When a ray of light from within a medium strikes its surface at an angle greater than the critical angle, the refracted ray is reflected back in the medium without emerging from that medium.
Study the illustration below.
> ${ }^{4}$ Note: The critical angle is the angle of incidence causing the refracted ray to emerge at $90^{\circ}$ to the normal (i.e. along the surface of the medium).


Total internal reflection is used in highway reflectors where maximum reflection is required. Also, in modern communication, total internal reflection is used in fiber optic cables.

1. Define total internal reflection and explain what is meant by the critical angle.

Total internal reflection refers to the bending of light as it travels from one medium into another with the refracted ray bending so much that it is reflected back into the same medium. The critical angle is that angle which causes the refrected ray to emerge along the surface $\left(\mathbf{9 0}^{\circ}\right)$.
2. The index of refraction of a particular liquid is 1.60. A ray of light inside the liquid strikes the liquid-air surface with an angle of incidence of $42^{\circ}$. Determine whether the ray will exit into the air or be totally reflected back into the liquid.

Find the critical angle $\left(\angle \mathrm{r}=90^{\circ}\right)$
$\mathrm{n}_{1} \operatorname{Sin} \mathrm{i}=\mathrm{n}_{2} \operatorname{Sin} \mathrm{r}$
$\therefore \operatorname{Sin} \mathrm{i}=\frac{\mathrm{n}_{2} \operatorname{Sin} \mathrm{r}}{\mathrm{n}_{1}}=\frac{(1)\left(\operatorname{Sin} 90^{\circ}\right)}{1.60}=0.625 \quad \therefore \mathrm{i}=38.6^{\circ}=39^{\circ}$
Since the given angle of incidence $\left(42^{\circ}\right)$ is greater than the critical angle $\left(39^{\circ}\right)$, the incident ray will not emerge from the liquid medium but instead will be reflected back int o the liquid.
3. In which medium does light travel faster, a medium with a critical angle of $25^{\circ}$ or a medium with a critical angle of $30^{\circ}$ ? Explain your answer.

Given the critical angles, we can calculate the index of refraction of each medium. The medium with the lower index of refraction has the faster speed of light.

$$
\mathrm{n}_{1} \operatorname{Sin} \mathrm{i}=\mathrm{n}_{2} \operatorname{Sin} \mathrm{r} \quad\left(\text { where } \mathrm{n}_{1}=1 \text { and } \mathrm{r}=90^{\circ}\right)
$$

$\therefore \quad \mathrm{n}_{2}=\frac{\mathrm{n}_{1} \operatorname{Sin} \mathrm{i}}{\operatorname{Sin} \mathrm{r}}=\frac{(1)\left(\operatorname{Sin} 25^{\circ}\right)}{\operatorname{Sin} 90^{\circ}}=0.423$ and $\mathrm{n}_{2}=\frac{\mathrm{n}_{1} \operatorname{Sin} \mathrm{i}}{\operatorname{Sin} \mathrm{r}}=\frac{(1)\left(\operatorname{Sin} 30^{\circ}\right)}{\operatorname{Sin} 90^{\circ}}=0.5$
Answer: Light travels faster in the medium having a critical angle of $25^{\circ}$.
4. A ray of light enters crown glass from air with an angle of incidence of $40^{\circ}$. Knowing that the angle of refraction is $25^{\circ}$, find the index of refraction of crown glass. [1.52]

$$
\begin{aligned}
& \because \mathrm{n}_{1} \operatorname{Sin} \mathrm{i}=\mathrm{n}_{2} \operatorname{Sin} \mathrm{r} \\
& \therefore \mathrm{n}_{2}=\frac{\mathrm{n}_{1} \operatorname{Sin} \mathrm{i}}{\operatorname{Sin} \mathrm{r}}=\frac{(1)\left(\operatorname{Sin} 40^{\circ}\right)}{\operatorname{Sin} 25^{\circ}}=1.52
\end{aligned}
$$

5. The index of refraction of crown glass is 1.52 . Calculate its critical angle. $\left[41^{\circ}\right]$

$$
\begin{aligned}
& n_{1} \operatorname{Sin} \mathrm{i}=\mathrm{n}_{2} \operatorname{Sin} \mathrm{r} \quad\left(\text { where } \angle \mathrm{r}=90^{\circ}\right) \\
\therefore & \operatorname{Sin} \mathrm{i}=\frac{\mathrm{n}_{2} \operatorname{Sin} \mathrm{r}}{\mathrm{n}_{1}}=\frac{(1)\left(\operatorname{Sin} 90^{\circ}\right)}{1.52}=0.6578 \quad \therefore \mathrm{i}=41.1^{\circ}=41^{\circ}
\end{aligned}
$$

6. A ray of light is incident upon an isosceles glass prism whose index of refraction is 1.50 . As shown in the diagram on the right, the angle of incidence is $38^{\circ}$ and the apex (top) angle of the prism is $50^{\circ}$. Calculate the angle of refraction. [ $41^{\circ}$ ]

## $24^{\circ}$ (Calculated using Snell's Law

Air-Glass
$\mathrm{n}_{1} \operatorname{Sin} \mathrm{i}=\mathrm{n}_{2} \operatorname{Sin} \mathrm{r} \quad \therefore \operatorname{Sin} \mathrm{r}=\frac{\mathrm{n}_{1} \operatorname{Sin} \mathrm{i}}{\mathrm{n}_{2}}=\frac{(1)\left(\operatorname{Sin} 38^{\circ}\right)}{1.5}=0.4104 \quad \therefore \mathrm{r}=24^{\circ}$
Glass - Air
$\mathrm{n}_{1} \operatorname{Sin} \mathrm{i}=\mathrm{n}_{2} \operatorname{Sin} \mathrm{r} \quad \therefore \operatorname{Sin} \mathrm{r}=\frac{\mathrm{n}_{1} \operatorname{Sin} \mathrm{i}}{\mathrm{n}_{2}}=\frac{(1.5)\left(\operatorname{Sin} 26^{\circ}\right)}{1}=0.6575 \quad \therefore \mathrm{r}=41^{\circ}$
7. The diagram below shows a beam of light traveling from water, through glass, to air. If the angle of incidence of the water-glass surface is $30^{\circ}$ (see diagram below), calculate the angle of refraction for the glass-air surface. [ $42^{\circ}$ ]


Water-Glass
$\mathrm{n}_{1} \operatorname{Sin} \mathrm{i}=\mathrm{n}_{2} \operatorname{Sin} \mathrm{r} \quad \therefore \operatorname{Sin} \mathrm{r}=\frac{\mathrm{n}_{1} \operatorname{Sin} \mathrm{i}}{\mathrm{n}_{2}}=\frac{(1.33)\left(\operatorname{Sin} 30^{\circ}\right)}{1.50}=0.4433 \quad \therefore \mathrm{r}=26.3^{\circ}$
Glass - Air
$\mathrm{n}_{1} \operatorname{Sin} \mathrm{i}=\mathrm{n}_{2} \operatorname{Sin} \mathrm{r} \quad \therefore \operatorname{Sin} \mathrm{r}=\frac{\mathrm{n}_{1} \operatorname{Sin} \mathrm{i}}{\mathrm{n}_{2}}=\frac{(1.50)\left(\operatorname{Sin} 26.3^{\circ}\right)}{1}=0.6646 \quad \therefore \mathrm{r}=42^{\circ}$
8. As illustrated in the diagram on the right, a ray of light is incident upon the surface of a medium at an angle of $60^{\circ}$ at which point both reflection and refraction occur. Knowing that the speed of light is one and one-half times faster in air than in this medium, find the angle formed by the refracted ray and the reflected ray. [85 ${ }^{\circ}$ ]


$$
\begin{aligned}
& \underline{\text { Index of refraction of the medium }: n}=\frac{c}{v_{n}}=\frac{3 \times 10^{8} \mathrm{~m} / \mathrm{s}}{\frac{3 \times 10^{8} \mathrm{~m} / \mathrm{s}}{1.5}}=1.5 \\
& \mathrm{n}_{1} \operatorname{Sin} \mathrm{i}=\mathrm{n}_{2} \operatorname{Sin} \mathrm{r} \quad \therefore \operatorname{Sir} \mathrm{r}=\frac{\mathrm{n}_{1} \operatorname{Sin} \mathrm{i}}{\mathrm{n}_{2}}=\frac{(1)\left(\operatorname{Sin} 60^{\circ}\right)}{1.5}=0.5773 \quad \therefore \mathrm{r}=35^{\circ} \\
& \text { ANSWER }: 30^{\circ}+\left(90^{\circ}-35^{\circ}\right)=85^{\circ}
\end{aligned}
$$

9. The speed of light in a clear plastic is $1.90 \times 10^{8} \mathrm{~m} / \mathrm{s}$. A ray of light enters the plastic at an angle of $38^{\circ}$. At what angle is the ray refracted? $\left[23^{\circ}\right]$

Find n for the plastic: $\mathrm{n}=\frac{\mathrm{c}}{\mathrm{v}_{\mathrm{n}}}=\frac{3 \times 10^{8} \mathrm{~m} / \mathrm{s}}{1.90 \times 10^{8} \mathrm{~m} / \mathrm{s}}=1.58$
$\because \mathrm{n}_{1} \operatorname{Sin} \mathrm{i}=\mathrm{n}_{2} \operatorname{Sin} \mathrm{r}$
$\therefore \operatorname{Sin} \mathrm{r}=\frac{\mathrm{n}_{1} \operatorname{Sin} \mathrm{i}}{\mathrm{n}_{2}}=\frac{(1)\left(\operatorname{Sin} 38^{\circ}\right)}{1.58}=0.3896 \quad \therefore \mathrm{r}=22.9^{\circ}=23^{\circ}$
10. At what angle of incidence should a ray of light approach the surface of diamond ( $\mathrm{n}=2.42$ ) from within the diamond so that the refracted ray emerges along the surface (parallel to the surface). [24 ${ }^{\circ}$ ]

$$
\begin{aligned}
& \mathrm{n}_{1} \operatorname{Sin} \mathrm{i}=\mathrm{n}_{2} \operatorname{Sin} \mathrm{r} \quad \text { where } \angle \mathrm{r}=90^{\circ} \\
\therefore & \operatorname{Sin} \mathrm{i}=\frac{\mathrm{n}_{2} \operatorname{Sin} \mathrm{r}}{\mathrm{n}_{1}}=\frac{(1)\left(\operatorname{Sin} 90^{\circ}\right)}{2.42}=0.4132 \quad \therefore \mathrm{i}=24.4^{\circ}=24^{\circ}
\end{aligned}
$$

11. A ray of light is incident at an angle of $40^{\circ}$ upon the water in an aquarium. As illustrated in the diagram on the right, the height of the water is 30 cm . Find the lateral displacement of the ray as it emerges the aquarium. [6.5]


Form $\triangle \mathrm{ABC}$ as shown. Then, at point C , draw a perpendicular to point D to form $\triangle \mathrm{ACD}$.
$\because \mathrm{n}_{1} \operatorname{Sin} \mathrm{i}=\mathrm{n}_{2} \operatorname{Sin} \mathrm{r} \quad \therefore \operatorname{Sin} \mathrm{r}=\frac{\mathrm{n}_{1} \operatorname{Sin} \mathrm{i}}{\mathrm{n}_{2}}=\frac{(1)\left(\operatorname{Sin} 40^{\circ}\right)}{1.33}=0.4832 \quad \therefore \mathrm{r}=28.8^{\circ}=29^{\circ}$
With reference to $\triangle \mathrm{ABC}: \quad \mathrm{AB}=30 \mathrm{~cm} \quad \angle \mathrm{BAC}=29^{\circ} \quad \mathrm{AC}=$ ?

$$
\because \operatorname{Cos} 29^{\circ}=\frac{30 \mathrm{~cm}}{\mathrm{AC}} \quad \therefore \mathrm{AC}=\frac{30 \mathrm{~cm}}{\operatorname{Cos} 29^{\circ}}=34.3 \mathrm{~cm}
$$

With reference to $\triangle \mathrm{ACD}: \quad \mathrm{AC}=34.3 \mathrm{~cm} \quad \angle \mathrm{CAD}=11^{\circ}\left(40^{\circ}-29^{\circ}\right) \quad \mathrm{CD}=$ ?

$$
\because \operatorname{Sin} 11^{\circ}=\frac{\mathrm{CD}}{34.3 \mathrm{~cm}} \quad \therefore \mathrm{CD}=(34.3 \mathrm{~cm})\left(\operatorname{Sin} 11^{\circ}\right)=6.5 \mathrm{~cm}
$$

